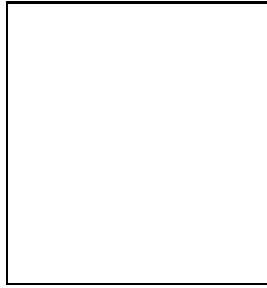


# EQUIVALENCE PRINCIPLE AND CLOCKS

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String theory suggests the existence of gravitational-strength scalar fields (“dilaton” and “moduli”) whose couplings to matter violate the equivalence principle. This provides a new motivation for high-precision clock experiments, as well as a generic theoretical framework for analyzing their significance.

## 1 Introduction

The equivalence principle was postulated by Einstein as a foundation stone for general relativity. The equivalence principle stipulates that the long-range gravitational interaction is entirely described by a universal coupling of “matter” (leptons, quarks, gauge fields and Higgs fields) to a (dynamical) second-rank symmetric tensor field  $g_{\mu\nu}(x^\lambda)$ , replacing everywhere in the matter Lagrangian the usual, kinematical, special relativistic (Minkowski) metric  $\eta_{\mu\nu}$ . This principle assumes that all the non-gravitational (dimensionless) coupling constants of matter (gauge couplings, CKM mixing angles, mass ratios, ...) are non-dynamical, i.e. take (at least at large distances) some fixed (vacuum expectation) values, independently of where and when, in spacetime, they are measured. Two of the best experimental tests of the equivalence principle are:

- (i) tests of the universality of free fall, i.e. the fact that all bodies fall with the same acceleration in an external gravitational field; and
- (ii) tests of the “constancy of the constants”.

Laboratory experiments (due notably, in our century, to Eötvös, Dicke, Braginsky and Adelberger) have verified the universality of free fall to the  $10^{-12}$  level. For instance, the fractional

difference in free fall acceleration of Beryllium and Copper samples was found to be <sup>1</sup>

$$\left(\frac{\Delta a}{a}\right)_{\text{Be Cu}} = (-1.9 \pm 2.5) \times 10^{-12}. \quad (1)$$

The Lunar Laser Ranging experiment <sup>2</sup> has also verified that the Moon and the Earth fall with the same acceleration toward the Sun to better than one part in  $10^{12}$

$$\left(\frac{\Delta a}{a}\right)_{\text{Moon Earth}} = (-3.2 \pm 4.6) \times 10^{-13}. \quad (2)$$

On the other hand, a recent reanalysis of the Oklo phenomenon (a natural fission reactor which operated two billion years ago in Gabon, Africa) gave a very tight limit on a possible time variation of the fine-structure “constant”, namely <sup>3</sup>

$$-0.9 \times 10^{-7} < \frac{e_{\text{Oklo}}^2 - e_{\text{now}}^2}{e^2} < 1.2 \times 10^{-7}, \quad (3)$$

$$-6.7 \times 10^{-17} \text{ yr}^{-1} < \frac{d}{dt} \ln e^2 < 5.0 \times 10^{-17} \text{ yr}^{-1}. \quad (4)$$

The tightness of the experimental limits (1)–(4) might suggest to apply Occam’s razor and to declare that the equivalence principle must be exactly enforced. However, the theoretical framework of modern unification theories, and notably string theory, suggest that the equivalence principle must be violated. Even more, the type of violation of the equivalence principle suggested by string theory is deeply woven into the basic fabric of this theory. Indeed, string theory is a very ambitious attempt at unifying all interactions within a consistent quantum framework. A deep consequence of string theory is that gravitational and gauge couplings are unified. In intuitive terms, while Einstein proposed a framework where geometry and gravitation were becoming united as a dynamical field  $g_{\mu\nu}(x)$ , i.e. a soft structure influenced by the presence of matter, string theory extends this idea by proposing a framework where geometry, gravitation, gauge couplings, and gravitational couplings all become soft structures described by interrelated dynamical fields. A symbolic equation expressing this softened, unified structure is

$$g_{\mu\nu}(x) \sim g^2(x) \sim G(x). \quad (5)$$

It is conceptually pleasing to note that string theory proposes to render dynamical the structures left rigid (or kinematical) by general relativity. Technically, Eq. (5) refers to the fact that string theory (as well as Kaluza-Klein theories) predicts the existence, at a fundamental level, of scalar partners of Einstein’s tensor field  $g_{\mu\nu}$ , the model-independent “dilaton” field  $\Phi(x)$ , and various “moduli fields”. The dilaton field, notably, plays a crucial role in string theory in that it determines the basic “string coupling constant”  $g_s = e^{\Phi(x)}$ , which determines in turn the (unified) gauge and gravitational coupling constants  $g \sim g_s$ ,  $G \propto g_s^2$ , as exemplified by the low-energy effective action

$$L_{\text{eff}} = e^{-2\Phi} \left[ \frac{R(g)}{\alpha'} + \frac{4}{\alpha'} (\nabla\Phi)^2 - \frac{1}{4} F_{\mu\nu}^2 - i\bar{\psi} D\psi - \dots \right]. \quad (6)$$

A softened structure of the type of Eq. (5), embodied in the effective action (6), implies a deep violation of Einstein’s equivalence principle. Bodies of different nuclear compositions fall with different accelerations because, for instance, the part of the mass of nucleus  $A$  linked to the Coulomb interaction of the protons depends on the space-variable fine-structure constant  $e^2(x)$  in a non-universal, composition-dependent manner. This raises the problem of the compatibility of the generic string prediction (5) with experimental tests of the equivalence principle, such as

Eqs. (1), (2) or (4). It is often assumed that the softness (5) applies only at short distances, because the dilaton and moduli fields are likely to acquire a non zero mass after supersymmetry breaking. However, a mechanism has been proposed<sup>4</sup> to reconcile in a natural manner the existence of a *massless* dilaton (or moduli) field as a fundamental partner of the graviton field  $g_{\mu\nu}$  with the current level of precision ( $\sim 10^{-12}$ ) of experimental tests of the equivalence principle. In the mechanism of<sup>4</sup> (see also<sup>5</sup> for metrically-coupled scalars) the very small couplings necessary to ensure a near universality of free fall,  $\Delta a/a < 10^{-12}$ , are dynamically generated by the expansion of the universe, and are compatible with couplings “of order unity” at a fundamental level.

The aim of the present paper is to emphasize the rich phenomenological consequences of long-range dilaton-like fields, and the fact that high-precision clock experiments might contribute to searching for, or constraining, their existence. More precisely, the basic question we wish to address here is the following: given the existing experimental tests of gravity, and given the currently favored theoretical framework, can high-precision clock experiments probe interesting theoretical possibilities which remain yet unconstrained? In addressing this question we wish to assume, as theoretical framework, the class of effective field theories suggested by string theory.

For historical completeness, let us mention that the theoretical framework which has been most considered in the phenomenology of gravitation, i.e. the class of “metric” theories of gravity<sup>6</sup>, which includes most notably the “Brans-Dicke”-type tensor-scalar theories, appears, from a modern perspective, as being rather artificial. This is good news because the phenomenology of “non metric” theories is richer and offers new possibilities for clock experiments. Historically, the restricted class of “metric” theories was introduced in 1956 by Fierz<sup>7</sup> to prevent, in an *ad hoc* way, too violent a conflict between experimental tests of the equivalence principle and the existence of a scalar contribution to gravity as suggested by the theories of Kaluza-Klein<sup>8</sup> and Jordan<sup>9</sup>. Indeed, Fierz was the first one to notice that a Kaluza-Klein scalar would generically strongly violate the equivalence principle. He then proposed to restrict artificially the couplings of the scalar field to matter so as to satisfy the equivalence principle. The restricted class of equivalence-principle-preserving couplings introduced by Fierz is now called “metric” couplings. Under the aegis of Dicke, Nordtvedt, Thorne and Will a lot of attention has been given to “metric” theories of gravity<sup>a</sup>, and notably to their quasi-stationary-weak-field phenomenology (“PPN framework”, see, e.g.,<sup>6</sup>).

## 2 Generic effective theory of a long-range dilaton

Motivated by string theory, we consider the generic class of theories containing a long-range dilaton-like scalar field  $\varphi$ . The effective Lagrangian describing these theories has the form (after a conformal transformation to the “Einstein frame”):

$$\begin{aligned}
L_{\text{eff}} = & \frac{1}{4q} R(g_{\mu\nu}) - \frac{1}{2q} (\nabla\varphi)^2 - \frac{1}{4e^2(\varphi)} (\nabla_\mu A_\nu - \nabla_\nu A_\mu)^2 \\
& - \sum_A \left[ \bar{\psi}_A \gamma^\mu (\nabla_\mu - iA_\mu) \psi_A + m_A(\varphi) \bar{\psi}_A \psi_A \right] + \dots
\end{aligned} \tag{7}$$

Here,  $q \equiv 4\pi \bar{G}$  where  $\bar{G}$  denotes a bare Newton’s constant,  $A_\mu$  is the electromagnetic field, and  $\psi_A$  a Dirac field describing some fermionic matter. At the low-energy, effective level (after the breaking of  $SU(2)$  and the confinement of colour), the coupling of the dilaton  $\varphi$  to matter is described by the  $\varphi$ -dependence of the fine-structure “constant”  $e^2(\varphi)$  and of the various masses  $m_A(\varphi)$ . Here,  $A$  is a label to distinguish various particles. [A deeper description would include

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<sup>a</sup>Note, however, that Nordtvedt, Will, Haugan and others (for references see<sup>6</sup>) studied conceivable phenomenological consequences of generic “non metric” couplings, without using a motivated field-theory framework to describe such couplings.

more coupling functions, e.g. describing the  $\varphi$ -dependences of the  $U(1)_Y$ ,  $SU(2)_L$  and  $SU(3)_c$  gauge coupling “constants”.]

The strength of the coupling of the dilaton  $\varphi$  to the mass  $m_A(\varphi)$  is given by the quantity

$$\alpha_A \equiv \frac{\partial \ln m_A(\varphi_0)}{\partial \varphi_0}, \quad (8)$$

where  $\varphi_0$  denotes the ambient value of  $\varphi(x)$  (vacuum expectation value of  $\varphi(x)$  around the mass  $m_A$ , as generated by external masses and cosmological history). For instance, the usual PPN parameter  $\gamma - 1$  measuring the existence of a (scalar) deviation from the pure tensor interaction of general relativity is given by<sup>10, 4</sup>

$$\gamma - 1 = -2 \frac{\alpha_{\text{had}}^2}{1 + \alpha_{\text{had}}^2}, \quad (9)$$

where  $\alpha_{\text{had}}$  is the (approximately universal) coupling (8) when  $A$  denotes any (mainly) hadronic object.

The Lagrangian (7) also predicts (as discussed in<sup>4</sup>) a link between the coupling strength (8) and the violation of the universality of free fall:

$$\frac{a_A - a_B}{\frac{1}{2}(a_A + a_B)} \simeq (\alpha_A - \alpha_B)\alpha_E \sim -5 \times 10^{-5} \alpha_{\text{had}}^2. \quad (10)$$

Here,  $A$  and  $B$  denote two masses falling toward an external mass  $E$  (e.g. the Earth), and the numerical factor  $-5 \times 10^{-5}$  corresponds to  $A = \text{Be}$  and  $B = \text{Cu}$ . The experimental limit Eq. (1) shows that the (mean hadronic) dilaton coupling strength is already known to be very small:

$$\alpha_{\text{had}}^2 \lesssim 10^{-7}. \quad (11)$$

Free fall experiments, such as Eq. (1) or the comparable Lunar Laser Ranging constraint Eq. (2), give the tightest constraints on any long-range dilaton-like coupling. Let us mention, for comparison, that solar-system measurements of the PPN parameters (as well as binary pulsar measurements) constrain the dilaton-hadron coupling to  $\alpha_{\text{had}}^2 < 10^{-3}$  (recently announced VLBI measurements improve this constraint to the  $2 \times 10^{-4}$  level), while the best current constraint on the time variation of the fine-structure “constant” (deduced from the Oklo phenomenon), namely Eq. (4), yields from Eq. (21) below,  $\alpha_{\text{had}}^2 \lesssim 3 \times 10^{-4}$ . [For an updated review of experimental tests of gravity, see the chapter 14 of the Review of Particle Physics, available on <http://pdg.lbl.gov/>]

To discuss the probing power of clock experiments, we need also to introduce other coupling strengths, such as

$$\alpha_{\text{EM}} \equiv \frac{\partial \ln e^2(\varphi_0)}{\partial \varphi_0}, \quad (12)$$

measuring the  $\varphi$ -variation of the electromagnetic (EM) coupling constant<sup>b</sup>, and

$$\alpha_A^{A*} \equiv \frac{\partial \ln E_A^{A*}(\varphi_0)}{\partial \varphi_0}, \quad (13)$$

where  $E_A^{A*}$  is the energy difference between two atomic energy levels.

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<sup>b</sup>Note that we do not use the traditional notation  $\alpha$  for the fine-structure constant  $e^2/4\pi\hbar c$ . We reserve the letter  $\alpha$  for denoting various dilaton-matter coupling strengths. Actually, the latter coupling strengths are analogue to  $e$  (rather than to  $e^2$ ), as witnessed by the fact that observable deviations from Einsteinian predictions are proportional to products of  $\alpha$ ’s, such as  $\alpha_A\alpha_E$ ,  $\alpha_{\text{had}}^2$ , etc...

In principle, the quantity  $\alpha_A^{A*}$  can be expressed in terms of more fundamental quantities such as the ones defined in Eqs. (8) and (12). For instance, in an hyperfine transition

$$E_A^{A*} \propto (m_e e^4) g_I \frac{m_e}{m_p} e^4 F_{\text{rel}}(Ze^2), \quad (14)$$

so that

$$\alpha_A^{A*} \simeq 2\alpha_e - \alpha_p + \alpha_{\text{EM}} \left( 4 + \frac{d \ln F_{\text{rel}}}{d \ln e^2} \right). \quad (15)$$

Here, the term  $F_{\text{rel}}(Ze^2)$  denotes the relativistic (Casimir) correction factor<sup>11</sup>. Moreover, in any theory incorporating gauge unification one expects to have the approximate link<sup>4</sup>

$$\alpha_A \simeq \left( 40.75 - \ln \frac{m_A}{1 \text{ GeV}} \right) \alpha_{\text{EM}}, \quad (16)$$

at least if  $m_A$  is mainly hadronic.

### 3 Clock experiments and dilaton couplings

The coupling parameters introduced above allow one to describe the deviations from general relativistic predictions in most clock experiments<sup>12</sup>. Let us only mention some simple cases.

First, it is useful to distinguish between “global” clock experiments where one compares spatially distant clocks, and “local” clock experiments where the clocks being compared are next to each other. The simplest global clock experiment is a static redshift experiment comparing (after transfer by electromagnetic links) the frequencies of the same transition  $A^* \rightarrow A$  generated in two different locations  $\mathbf{r}_1$  and  $\mathbf{r}_2$ . The theory of Section 2 predicts a redshift of the form (we use units in which  $c = 1$ )

$$\frac{\nu_A^{A*}(\mathbf{r}_1)}{\nu_A^{A*}(\mathbf{r}_2)} \simeq 1 + (1 + \alpha_A^{A*} \alpha_E) (\overline{U}_E(\mathbf{r}_2) - \overline{U}_E(\mathbf{r}_1)), \quad (17)$$

where

$$\overline{U}_E(\mathbf{r}) = \frac{\overline{G} m_E}{r} \quad (18)$$

is the *bare* Newtonian potential generated by the external mass  $m_E$  (say, the Earth). Such a result has the theoretical disadvantage of depending on other experiments for its interpretation. Indeed, the *bare* potential  $\overline{U}_E$  is not directly measurable. The measurement of the Earth potential by the motion of a certain mass  $m_B$  gives access to  $(1 + \alpha_B \alpha_E) \overline{U}_E(\mathbf{r})$ . The theoretical significance of a global clock experiment such as (17) is therefore fairly indirect, and involves other experiments and other dilaton couplings. One can generalize (17) to a more general, non static experiment in which different clocks in relative motion are compared. Many different “gravitational potentials” will enter the result, making the theoretical significance even more involved.

A conceptually simpler (and, probably, technologically less demanding) type of experiment is a differential, “local” clock experiment. Such “null” clock experiments have been proposed by Will<sup>6</sup> and first performed by Turneure et al.<sup>13</sup>. The theoretical significance of such experiments within the context of dilaton theories is much simpler than that of global experiments. For instance if (following the suggestion of<sup>14</sup>) one locally compares two clocks based on hyperfine transitions in alkali atoms with different atomic number  $Z$ , one expects to find a ratio of frequencies

$$\frac{\nu_A^{A*}(\mathbf{r})}{\nu_B^{B*}(\mathbf{r})} \simeq \frac{F_{\text{rel}}(Z_A e^2(\varphi_{\text{loc}}))}{F_{\text{rel}}(Z_B e^2(\varphi_{\text{loc}}))}, \quad (19)$$

where the local, ambient value of the dilaton field  $\varphi_{\text{loc}}$  might vary because of the (relative) motion of external masses with respect to the clocks (including the effect of the cosmological expansion). The directly observable fractional variation of the ratio (19) will consist of two factors:

$$\delta \ln \frac{\nu_A^{A*}}{\nu_B^{B*}} = \left[ \frac{\partial \ln F_{\text{rel}}(Z_A e^2)}{\partial \ln e^2} - \frac{\partial \ln F_{\text{rel}}(Z_B e^2)}{\partial \ln e^2} \right] \times \delta \ln e^2. \quad (20)$$

The “sensitivity” factor in brackets due to the  $Z$ -dependence of the Casimir term can be made of order unity<sup>14</sup>, while the fractional variation of the fine-structure constant is expected in dilaton theories to be of order<sup>4, 12</sup>

$$\begin{aligned} \delta \ln e^2(t) &= -2.5 \times 10^{-2} \alpha_{\text{had}}^2 U(t) \\ &- 4.7 \times 10^{-3} \kappa^{-1/2} (\tan \theta_0) \alpha_{\text{had}}^2 H_0(t - t_0). \end{aligned} \quad (21)$$

Here,  $U(t)$  is the value of the externally generated gravitational potential at the location of the clocks, and  $H_0 \simeq 0.5 \times 10^{-10} \text{ yr}^{-1}$  is the Hubble rate of expansion. [The factor  $\kappa^{-1/2} \tan \theta_0$  is expected to be  $\sim 1$ .]

The (rough) theoretical prediction (21) allows one to compare quantitatively the probing power of clock experiments to that of equivalence principle tests. Let us (optimistically) assume that clock stabilities of order  $\delta\nu/\nu \sim 10^{-17}$  (for the relevant time scale) can be achieved. A differential *ground* experiment (using the variation of the Sun’s potential due to the Earth eccentricity) would probe the level  $\alpha_{\text{had}}^2 \sim 3 \times 10^{-6}$ . A geocentric satellite differential experiment could probe  $\alpha_{\text{had}}^2 \sim 5 \times 10^{-7}$ . These levels are impressive (compared to present solar-system tests of the PPN parameter  $\gamma$  giving the constraint  $\alpha_{\text{had}}^2 \simeq (1 - \gamma)/2 < 2 \times 10^{-4}$ ), but are not as good as the present equivalence-principle limit (11). To beat the level (11) one needs to envisage an heliocentric differential clock experiment (a few solar radii probe within which two hyper-stable clocks are compared). Such an experiment could, according to Eq. (21), reach the level  $\alpha_{\text{had}}^2 \sim 10^{-9}$ . [Let us also note that a gravitational time delay global experiment using clocks beyond the Sun as proposed by C. Veillet (SORT concept) might (optimistically) probe the level  $\alpha_{\text{had}}^2 \sim 10^{-7}$ .] It is, however, to be noted that a much refined test of the equivalence principle such as STEP (Satellite Test of the Equivalence Principle) aims at measuring  $\Delta a/a \sim 10^{-18}$  which corresponds to the level  $\alpha_{\text{had}}^2 \sim 10^{-14}$ , i.e. five orders of magnitude better than any conceivable clock experiment.

## 4 Conclusions

In summary, the main points of the present contribution are:

- Independently of any theory, the result (4) of a recent reanalysis of the Oklo phenomenon<sup>3</sup> gives a motivation, and a target, for improving laboratory clock tests of the time variation of the fine-structure constant  $e^2$  (which are at the  $3.7 \times 10^{-14} \text{ yr}^{-1}$  level<sup>14</sup>).
- Modern unification theories, and especially string theory, suggest the existence of new gravitational-strength fields, notably scalar ones (“dilaton” or “moduli”), whose couplings to matter violate the equivalence principle. These fields would induce a spacetime variability of the coupling constants of physics (such as the fine-structure constant). High-precision clock experiments are excellent probes of such a possibility.
- The generic class of dilaton theories defined in Section 2 provides a well-defined theoretical framework in which one can discuss the phenomenological consequences of the existence of a dilaton-like field. Such a theoretical framework (together with some assumptions, e.g. about gauge unification and the origin of mass hierarchy) allows one to compare and contrast the probing power of clock experiments to that of other experiments.

- Local, differential clock experiments (of the “null” type of <sup>13</sup>) appear as conceptually cleaner, and technologically less demanding, probes of dilaton-motivated violations of the equivalence principle than global, absolute clock experiments (of the Gravity Probe A type).
- If we use the theoretical assumptions of Section 2 to compare clock experiments to free-fall experiments, one finds that one needs to send and intercompare two ultra-high-stability clocks in near-solar orbit in order to probe dilaton-like theories more deeply than *present* free-fall experiments. Currently proposed improved satellite tests of the equivalence principle would, however, beat any clock experiment in probing even more deeply such theories.
- At the qualitative level, it is, however, important to note that clock experiments (especially of the “global”, GPA type) probe different combinations of basic coupling parameters than free-fall experiments. This is visible in Eq. (15) which shows that  $\alpha_A^{A*}$  contains the leptonic quantity  $\alpha_e = \partial \ln m_{\text{electron}} / \partial \varphi_0$  without any small factor<sup>c</sup>.

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<sup>c</sup>Free-fall experiments couple predominantly to hadronic quantities such as  $\alpha_p = \partial \ln m_{\text{proton}} / \partial \varphi_0$ , and to Coulomb-energy effects proportional to  $\alpha_{\text{EM}}$ . The effect of the leptonic quantity  $\alpha_e$  is down by a small factor  $\sim m_e/m_p \sim 1/1836$ .